

Issues from P4

Which sections of the specification do students need to know to do the question?

Is the question suitable (with rigorous wording etc) for a written examination.

Is the question better suited to classroom use?

Avoid this question!

**1.** Prove by contradiction that  $\sqrt{3}$  is irrational.

**2** If I use the same sort of process to 'prove'  $\sqrt{4}$  is irrational where does the process go wrong?

**3** Prove that  $\sqrt[3]{3}$  is irrational.

**4.** Suppose  $k$  is a rational number and  $n$  is an irrational number.

(a) Prove by contradiction that  $k + n$  and  $kn$  are irrational.

Suppose both  $k$  and  $n$  are irrational.

(b) What can be concluded then?

**5** Prove, by contradiction, that no power of 2 can be written as the sum of consecutive numbers.

**6** Use a similar approach to that of finding  $\int e^{2x} \cos x dx$  to find  $\int \sec^4 x dx$

**7** Make a critique of this proof of the irrationality of  $\sqrt{2}$

Start with  $(\sqrt{2} - 1)(\sqrt{2} + 1) = 1 \Rightarrow \sqrt{2} = \frac{1}{\sqrt{2} - 1} - 1$

Now let  $\sqrt{2} = \frac{a}{b}$  where  $a, b$  are whole numbers in their lowest terms and  $2b > a > b > 1$

So  $\sqrt{2} = \frac{1}{\sqrt{2} - 1} - 1 \Rightarrow \sqrt{2} = \frac{1}{\frac{a}{b} - 1} - 1 = \frac{2b - a}{a - b}$

Now  $2b - a$  and  $a - b$  are positive whole numbers but  $a - b < b$  and  $2b - a < a$  so we have a new denominator and numerator .....